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A GENERAL CLASS OF ESTIMATORS UNDER MULTI PHASE SAMPLING

M. S. Ahmed[1] & Atsu S. S. Dorvlo[2]

ABSTRACT

This paper derives the general estimators for finite population mean using multivariate auxiliary information under multiphase sampling. Here a number of auxiliary variables are considered in each phase under general sampling design. The properties of these estimators are studied and the results are presented for simple random sampling without replacement (SRSWOR) scheme. Using a modified cost function the optimum sample sizes are also derived.

Key words and Phrases: Multivariate auxiliary information; Multiphase sampling; Chain estimators.

1. Introduction

Multi-phase sampling scheme is very useful for estimating finite population parameters when a number of auxiliary variables are available cheaply related to the survey variable. The estimation of population mean of a survey variable under the partial knowledge of the auxiliary means has been considered by Chand (1975), Kiregyera (1980, 1984), Mukerjee *et al*. (1987) and Srivastava *et al*. (1990). However, their results are confined to the use of two auxiliary variables only, the mean of one is known while the other is unknown. They considered the two-phase sampling scheme only for simple random sampling without replacement (SRSWOR) at both the phases. Ahmed (1995) and Tripathi & Ahmed (1995) extended these results considering more than two auxiliary variables in two-phase sampling. Some recent works related to multiphase sampling are Ahmed *et al*. (1995&96), Ahmed (2003), and Diana *et al*.2004).

Suppose that $S_0 = (S_{01}, S_{02}, ..., S_{0n_0})$ is a finite population of size n_0 (given) and Y denotes the study variable with population mean $\overline{Y}_{(0)}$ ($\overline{Y}_{(0)} = \sum_{S_0} Y_i / n_0$).

[1] M S Ahmed, email: msahmed@squ.edu.om (Corresponding author).
[2] Atsu S. S. Dorvlo, email: atsu@squ.edu.om, Department of Mathematics and Statistics, Sultan Qaboos University, P.O. Box 36, Al-Khoudh PC 123, Muscat, Sultanate of Oman.

Suppose that $\underline{X}_i^{(r)} = (X_i^{(0)\prime}, X_i^{(1)\prime}, X_i^{(2)\prime}, ..., X_i^{(r)\prime})'$, where $X_i^{(r)} = (X_{1i}, X_{2i} ..., X_{k_r i})'$ are k-auxiliary variables ($k = \sum_{r=0}^{m} k_r$) and they are available from r-th phase source (r=1,2,...,m) with moderate cost to estimate the population mean $\overline{Y}_{(0)}$ of the study variable Y.

Suppose that X_{jr} is available for all $j \in S_{r-1}$, where S_r is a sub-sample drawn from S_{r-1} under the sampling design D_r (r = 1,2,...,m). The study variable Y_i is observed for all $i \in S_m$, a comparatively small sample with moderate cost.

Suppose that $\overline{Y}_{(m)}$ is an unbiased estimate of the population mean $\overline{Y}_{(0)}$ under the sampling design D_m at the m-th phase. Further, $\overline{X}_{j(r)}$ is an unbiased estimate of the population mean $\overline{X}_{j(0)}$ ($\overline{X}_{j(0)} = \sum_{S_0} X_{ji} / n_0$ under the sampling design D_r (r = 1,2,...,m) at the r-th phase.

The layout of auxiliary variables and study variable for generalized multiphase sampling are given as follows:

Source		Size	Auxiliary Variables	No. of variables
Population	S_0	n_0	$(X_{01},\ X_{02}, \ldots, X_{0k_0})' = \underline{X}_0$	k_0
1st phase	S_1	n_1	$(X_{11},\ X_{12}, \ldots, X_{1k_1})' = \underline{X}_1$	k_1
2nd phase	S_2	n_2	$(X_{21},\ X_{22}, \ldots,\ X_{2k_2})' = \underline{X}_2$	k_2
…	…	…	…	…
m-1th phase	S_{m-1}	n_{m-1}	$(X_{m-11}, X_{m-12}, \ldots, X_{m-1k_m})' = \underline{X}_{m-1}$	k_{m-1}

$n_i \le n_{i-1}$

Total auxiliary variables gather	Size	No. of variables
$\underline{X}_0 = \underline{X}^{(0)}$	n_0	k_0
$(\underline{X}_1,\ \underline{X}_0)' = \underline{X}^{(1)}$	n_1	$k_0 + k_1$
$(\underline{X}_0,\ \underline{X}_1,\ \underline{X}_2)' = \underline{X}^{(2)}$	n_2	$k_0 + k_1 + k_2$
	…	
$(\underline{X}_0,\ \underline{X}_1,\ \underline{X}_2, \ldots, \underline{X}_i)' = \underline{X}^{(i)}$	n_i	$k_0 + k_1 + k_2 + \ldots + k_i$
…		…
$(\underline{X}_0,\ \underline{X}_1,\ \underline{X}_2, \ldots, \underline{X}_i, \ldots, \underline{X}_{m-1})' = \underline{X}^{(m-1)}$	n_{m-1}	$k_0 + k_1 + k_2 + \ldots + k_{m-1}$
$(\underline{X}_0,\ \underline{X}_1,\ \underline{X}_2, \ldots, \underline{X}_i, \ldots, \underline{X}_m)' = \underline{X}^{(m)}$	n_m	$k_0 + k_1 + k_2 + \ldots + k_m$

Some particular cases of generalized multiphase sampling are given as follows:

k_0=1 and $k_1 = k_2 = ... = k_{m-1} = 0$ Classical Ratio and Regression estimators	Cochran (1977)
k_0>1 and $k_1 = k_2 = ... = k_{m-1} = 0$ Multiple Ratio and Regression estimators	Cochran (1977)
k_1=1 and $k_0 = k_2 = k_3 = ... = k_{m-1} = 0$ Classical Two-phase or Double Sampling Ratio and Regression estimators	Cochran (1977)
k_1>1 and $k_0 = k_2 = k_3 = ... = k_{m-1} = 0$ Multivariate Two-phase or Double Sampling Ratio and Regression estimators	Cochran (1977)
$k_0 = k_1$=1 and $k_2 = k_3 = ... = k_{m-1} = 0$ Two-phase or Double Sampling Ratio and Regression estimators with two auxiliary variables	Chand (1975), Kiregyera (1980,1984), Srivastava *et al.* (1990), Mukerjee *et al.* (1987), Singh *et al.* (1994), Ahmed and Ali (1996), Ahmed (1997), Ahmed *et al.* (1998).
$k_0 > 1$, k_1>1 and $k_2 = k_3 = ... = k_{m-1} = 0$ Two-phase or Double Sampling Ratio and Regression estimators with more auxiliary variables	Ahmed *et al.* (1994) and Tripathi and Ahmed (1995).
$k_1 = k_2 = ... = k_{m-1} = 1$ Multiphase Ratio and Regression estimators with more auxiliary variables	Diana *et al.* (2004)
$k_1 \geq 1, k_2 \geq 1 ... = k_{m-1} \geq 1$ Multiphase Ratio and Regression estimators with more auxiliary variables	Ahmed *et al*. (1995&1996), Ahmed (2003)

2. The proposed estimators and their properties

For estimating $\overline{Y}_{(0)}$ we may consider the estimation procedure,

$$G_m = g\left(\overline{Y}_{(m)}, \underline{\overline{X}}^{(0)}_{(0)}, \underline{\overline{X}}^{(0)}_{(1)}, \underline{\overline{X}}^{(1)}_{(1)}, \underline{\overline{X}}^{(1)}_{(2)}, \ldots, \underline{\overline{X}}^{(m-1)}_{(m-1)} \underline{\overline{X}}^{(m-1)}_{(m)}\right)$$

$$G_m = g\left(\underbrace{\underbrace{\underbrace{\bar{Y}_{(m)}, \overbrace{\underline{\bar{X}}^{(0)}_{(0)}, \underline{\bar{X}}^{(0)}_{(1)}}^{k_0}}_{1st}, \overbrace{\underline{\bar{X}}^{(1)}_{(1)}, \underline{\bar{X}}^{(1)}_{(2)}}^{k_0+k_1}}_{2nd}, \ldots, \overbrace{\underline{\bar{X}}^{(m-1)}_{(m-1)} \underline{\bar{X}}^{(m-1)}_{(m)}}^{k_0+k_1+\ldots+k_{m-1}}}_{m-th}\right) \quad m>0 \tag{2.1}$$

$$H_m = \bar{Y}_{(m)} h\left(\underline{\bar{X}}^{(0)}_{(0)}, \underline{\bar{X}}^{(0)}_{(1)}, \underline{\bar{X}}^{(1)}_{(1)}, \underline{\bar{X}}^{(1)}_{(2)}, \ldots, \underline{\bar{X}}^{(m-1)}_{(m-1)} \underline{\bar{X}}^{(m-1)}_{(m)}\right) \tag{2.2}$$

Using linearization, Ahmed *et al*. (1995&1996) showed that

$$G_m \approx H_m \approx \bar{Y}_{(m)} + \sum_{r=1}^{m} (\underline{\bar{X}}^{(r-1)}_{(r-1)} - \underline{\bar{X}}^{(r-1)}_{(r)})' \underline{\boldsymbol{T}}^{(r)} \tag{2.3}$$

Where $\underline{\boldsymbol{T}}^{(r)} = \left(\underline{T}^{(0)\prime}, \underline{T}^{(1)\prime}, \ldots, \underline{T}^{(r)\prime}\right)'$ and $\underline{T}^{(r)} = \left(T_1^{(r)}, T_2^{(r)}, \ldots, T_{k_0}^{(r)}\right)'$

Two special cases of H_m as Ahmed (2003) defined

$$H_m^{(1)} = \bar{Y}_{(m)} \cdot \prod_{r=1}^{m} \prod_{j=1}^{k_r} \left(\frac{\bar{X}_{j(r-1)}}{\bar{X}_{j(r)}}\right)^{a_{j(r)}} ; \bar{X}_{j(0)} = \bar{X}_j \tag{2.4}$$

Another estimation procedure based on fixed weights defined as

$$H_m^{(2)} = \bar{Y}_{(m)} \cdot \prod_{r=1}^{m} \prod_{j=1}^{k_r} u_{j(r)} \left(\frac{\bar{X}_{j(r-1)}}{\bar{X}_{j(r)}}\right)^{a_{j(r)}} ; \bar{X}_{j(0)} = \bar{X}_j \tag{2.5}$$

where $a_{j(r)}$ are suitably chosen constants and $u_{j(r)}$ are fixed weights.

We assume that $\bar{X}_{j(r)}$ is defined such as to be conditionally unbiased for $\bar{X}_{j(r)}$ i.e. the conditional expectation given S_{r-1}

$$E_r\left(\bar{X}_{j(r)}\right) = \bar{X}_{j(r-1)} \quad \text{for all } j=1,2,\ldots,k_r \text{ and } r=1,2,\ldots,m \tag{2.6}$$

Further we have the conditional covariance given S_r

$$C_r\left(\bar{X}_{j(r)} \bar{X}_{j(r-1)}\right) = 0 \text{ and } C_r\left(\bar{X}_{j(r)} \bar{Y}_{(r-1)}\right) = 0 \tag{2.7}$$

where E_r and C_r stand for conditional expectation and covariance respectively at r-th phase given S_{r-1}, $r = 1,2, \ldots ,m$.

It is always possible to define unbiased estimates $\bar{X}_{j(r)}$ and $\bar{Y}_{(m)}$ provided each $S_{0i} \in S_0$, $S_0 = (S_{01}, S_{02}, \ldots, S_{0n_0})$ has possible probability of selecting for fixed size sampling scheme. If $\pi_1^{(r)}$, $\pi_2^{(r)}$, ... , $\pi_{n_0}^{(r)}$; $\pi_1^{(r)} > 0$ for all $S_{0i} \in S_0$, ($\sum_{S_0} \pi_i^{(r)} = n_r$) are the inclusion probabilities for S_r, where $\pi_i^{(r)} = \pi_i^{(r-1)} . \pi_{i|S_{(r-1)}}^{(r-1)}$. Here $\pi_{i|S_{(r-1)}}^{(r-1)}$ is the conditional inclusion probability of i-th unit in S_r given S_{r-1} then

$$\overline{X}_{j(r)} = \frac{1}{n_r}\sum_{S_r}\frac{X_{j(r)}}{\pi_i^{(r)}} \text{ and } \overline{Y}_{(r)} = \frac{1}{n_r}\sum_{S_r}\frac{Y_i}{\pi_i^{(r)}} \tag{2.8}$$

Define,

$R_j = \overline{Y}_{(0)} / \overline{X}_{j(0)}$, $m_{jj'}^{(r)} = E_1 E_2 ... E_{r-1} C_r \left(\overline{X}_{j(r)}, \overline{X}_{j'(r)}\right)$

and $\alpha_j^{(r)} = E_1 E_2 .... E_{r-1} C_r \left(\overline{X}_{j(r)}, \overline{Y}_{(r)}\right)$.

Define,

$a^{(r)} = \left(a_1^{(r)}, a_2^{(r)}, ..., a_{k_r}^{(r)}\right)'$, $\underline{a}^{(r)} = diag\left(a_1^{(r)}, a_2^{(r)}, ..., a_{k_r}^{(r)}\right)$,

$b^{(r)} = \left(b_1^{(r)}, b_2^{(r)}, ..., b_{k_r}^{(r)}\right)'$, $\underline{b}^{(r)} = diag\left(b_1^{(r)}, b_2^{(r)}, ..., b_{k_r}^{(r)}\right)$,

$u^{(r)} = \left(u_1^{(r)}, u_2^{(r)}, ..., u_{k_r}^{(r)}\right)'$, $\underline{u}^{(r)} = diag\left(u_1^{(r)}, u_2^{(r)}, ..., u_{k_r}^{(r)}\right)$,

$\underline{R}^{(r)} = diag\left(R_1^{(r)}, R_2^{(r)}, ..., R_{k_r}^{(r)}\right)$, $\alpha^{(r)} = \left(\alpha_1^{(r)}, \alpha_2^{(r)}, ..., \alpha_{k_r}^{(r)}\right)'$, $m^{(r)} = \left(m_{jj'}^{(r)}\right)$

and $m_{(r)}^{-1} = \left(m_{(r)}^{jj'}\right)$; $\forall j, j' = 1,2,...,k_r$.

For large samples, the contribution of the third and higher order products and central moments can be ignored. Taking the expectation both sides and after some simplification, we have stated the following theorems.

Theorem 2.1: For large samples, the biases and mean square errors of G_1, H_1 and H_2 for estimating the population mean of the study variable Y are given respectively by,

$$M\left(G_1\right) = V\left(\overline{Y}_{(m)}\right) + \sum_{r=1}^{m} \underline{\boldsymbol{T}}^{(r)'}\left(m^{(r)}\underline{\boldsymbol{T}}^{(r)} - 2\alpha^{(r)}\right) \tag{2.9}$$

$$B\left(H_1\right) = \frac{1}{2\overline{Y}}\sum_{r=1}^{m}\Big(\underline{a}^{(r)}\underline{R}^{(r)}m^{(r)}\underline{R}^{(r)}\underline{a}^{(r)} + trace\ \underline{a}^{(r)}\underline{R}^{(r)}m^{(r)}\underline{R}^{(r)} - 2\underline{a}^{(r)}\underline{R}^{(r)}\underline{a}^{(r)}\ \Big) \tag{2.10}$$

$$M\left(H_1\right) = V\left(\overline{Y}_{(m)}\right) + \sum_{r=1}^{m}\underline{a}^{(r)}\underline{R}^{(r)}\left(m^{(r)}\underline{R}^{(r)}\underline{a}^{(r)} - 2\alpha^{(r)}\right) \tag{2.11}$$

$$B(H_2)=\frac{1}{2\bar{Y}}\sum_{r=1}^{m}\Big(\underline{b}^{(r)}\underline{R}^{(r)}m^{(r)}\underline{R}^{(r)}\underline{b}^{(r)}\underline{u}^{(r)}+trace\,\underline{b}^{(r)}\underline{R}^{(r)}m^{(r)}\underline{R}^{(r)}\underline{u}^{(r)}$$

$$-2\underline{b}^{(r)}\underline{u}^{(r)}\underline{R}^{(r)}\alpha^{(r)}\Big) \qquad (2.12)$$

$$M(H_2)=V\left(\bar{Y}_{(m)}\right)+\sum_{r=1}^{m}\underline{b}^{(r)}\underline{u}^{(r)}\underline{R}^{(r)}\left(m^{(r)}\underline{R}^{(r)}\underline{u}^{(r)}\underline{b}^{(r)}-2\alpha^{(r)}\right) \qquad (2.13)$$

Theorem 2.2: The optimum choices of $\underline{\boldsymbol{T}}^{(r)}$, $a^{(r)}$ and $b^{(r)}$ which minimizes $M(G_1)$, $M(H_1)$ and $M(H_2)$ are given respectively by $\underline{\boldsymbol{T}}^{(r)}=m^{(r)^{-1}}\alpha^{(r)}$, $a_0^{(r)}=\underline{R}^{(r)^{-1}}m^{(r)^{-1}}\alpha^{(r)}$ and $b_0^{(r)}=\underline{R}^{(r)^{-1}}m^{(r)^{-1}}\underline{u}^{(r)^{-1}}\alpha^{(r)}$ for all r and for large samples, the minimum mean square errors of G_1, H_1 and H_2 are same and given by

$$M_0=V\left(\bar{Y}_{(m)}\right)-\sum_{r=1}^{m}\alpha^{(r)'}m^{(r)^{-1}}\alpha^{(r)} \qquad (2.14)$$

3. Results under SRSWOR scheme

Now, we will give the results when the SRSWOR scheme is adopted for selecting S_r for all $r = 1,2,...,m$.

Define,

$$\bar{X}_{j(r)}=\frac{1}{n_r}\sum\nolimits_{S_r}X_{ji},\ \bar{Y}_{(r)}=\frac{1}{n_r}\sum\nolimits_{S_r}Y_i,\ \sigma_{jj'}=\frac{1}{n_0}\sum\nolimits_{U}\left(X_{ji}-\bar{X}_j\right)\left(X_{j'i}-\bar{X}_{j'}\right)$$

$$\sigma_{jy}=\frac{1}{n_0}\sum\nolimits_{U}\left(X_{ji}-\bar{X}_j\right)\left(Y_i-\bar{Y}\right) \text{ and } \sigma_y^2=\frac{1}{n_0}\sum\nolimits_{U}\left(Y_i-\bar{Y}\right)^2$$

for all $j,j'=1,2,....,k_r$ and $r = 0, 1,2,....,m$.

Also define $\Lambda=\begin{pmatrix}\Lambda_{11} & \Lambda_{12} & ... & \Lambda_{1m}\\ \Lambda'_{12} & \Lambda_{22} & ... & \Lambda_{2m}\\ .. & .. & ... & ..\\ \Lambda'_{1m} & \Lambda'_{2m} & ... & \Lambda_{mm}\end{pmatrix}$, $\Lambda_{rr'}=\left(\sigma_{jj'}\right)$,

$\phi=\left(\phi'_{(1)},\phi'_{(2)},....,\phi'_{(r)}\right)'=\left(\sigma_{jy}\right)$ and $\phi_{(r)}=\left(\sigma_{jy}\right)$ for all $j,j'=1,2,....,k_r$ and $r = 1,2,....,m$.

If SRSWOR is used at all the phases then

$\alpha^{(r)}=n(r,r-1)\phi_{(r)}$, $m^{(r)}=n(r,r-1)\Lambda_{rr'}$ and

$$V\left(\overline{Y}_{(m)}\right) = n(r, r-1)\sigma_y^2 \tag{3.1}$$

where $n(a,b) = \left(\dfrac{n_0}{n_0 - 1}\right)\left(\dfrac{1}{n_a} - \dfrac{1}{n_b}\right)$

For SRSWOR, the biases and mean square errors of T_1 and T_2 are respectively, given by

$$B\left(H_1\right) = \frac{1}{2\overline{Y}}\sum_{r=1}^{m} n(r, r-1)(\underline{a}^{(r)}\underline{R}^{(r)}\Lambda_{rr'}\underline{R}^{(r)}\underline{a}^{(r)} + Trace\ \underline{a}^{(r)}\underline{R}^{(r)}\Lambda_{rr'}\underline{R}^{(r)}$$

$$-2\underline{a}^{(r)}\underline{R}^{(r)}\underline{a}^{(r)}) \tag{3.2}$$

$$M\left(H_1\right) = n(m,0)\sigma_y^2 + \sum_{r=1}^{m} n(r, r-1)\underline{a}^{(r)}\underline{R}^{(r)}\left(\Lambda_{rr'}\underline{R}^{(r)}\underline{a}^{(r)} - 2\phi_{(r)}\right) \tag{3.3}$$

$$M\left(G_1\right) = n(m,0)\sigma_y^2 + \sum_{r=1}^{m} n(r, r-1)\underline{\boldsymbol{T}}^{(r)}\left(\Lambda_{rr'}\underline{\boldsymbol{T}}^{(r)} - 2\phi_{(r)}\right) \tag{3.3}$$

$$B\left(H_2\right) = \frac{1}{2\overline{Y}}\sum_{r=1}^{m} n(r, r-1)(\underline{b}^{(r)}\underline{R}^{(r)}\Lambda_r\underline{R}^{(r)}\underline{b}^{(r)}\underline{u}^{(r)} + Trace\ \underline{b}^{(r)}\underline{R}^{(r)}\Lambda_r\underline{R}^{(r)}\underline{u}^{(r)}$$

$$-\ 2\underline{b}^{(r)}\underline{u}^{(r)}\underline{R}^{(r)}\phi_{(r)} \tag{3.4}$$

$$M\left(H_2\right) = n(m,0)\sigma_y^2 + \sum_{r=1}^{m} n(r, r-1)\underline{b}^{(r)'}\underline{u}^{(r)}\underline{R}^{(r)}(\Lambda_{r'r}\underline{R}^{(r)}\underline{u}^{(r)}b^{(r)} - 2\phi_{(r)}) \tag{3.5}$$

For SRSWOR, the optimum choices of $\underline{\boldsymbol{T}}^{(r)}$, $a^{(r)}$ and $b^{(r)}$ which minimize $M(G_1)$, $M(H_1)$ and $M(H_2)$ are given respectively by

$\underline{\boldsymbol{T}}_{\boldsymbol{0}}^{(r)} = \Lambda_{r'r}^{-1}\phi_{(r)}$, $a_0^{(r)} = \underline{R}^{(r)^{-1}}\Lambda_{r'r}^{-1}\phi_{(r)}$ and $b_0^{(r)} = \underline{R}^{(r)^{-1}}\Lambda_{r'r}^{-1}\underline{u}^{(r)^{-1}}\phi_{(r)}$ for all $r, r' = 1,2,...,m$

For large samples, the minimum mean square error of H_1 and H_2 are same and given by

$$M\left(H_1\right)_{\min} = \sigma_y^2\left[n(m,0)\left(1 - \rho_{y.12....k_m}^2\right) + \sum_{r=1}^{m} n(r, r-1)\left(1 - \rho_{y.12....k_{r-1}}^2\right)\right] \tag{3.6}$$

where $\rho_{y.12....k_r}$ is multiple correlation coefficient.

4. Selection of optimum sample sizes for a fixed cost

For designing a sample survey efficiently, it is essential to have some broad information about the variability in the population and on the cost of different steps involved in carrying out the survey. A design-based measure of the sampling error in the results of the survey is given by the mean square error of the estimator used, which reduces to its variance in the case of an unbiased estimator. In sample surveys, the variance of the estimator usually decreases with increase in sample size. Further, variance and cost would also depend on the nature of the sampling unit. Hence, for the above sampling scheme it becomes necessary to take both these aspects into account in arriving at the optimum sampling unit and the optimum sizes n_r which would provide the maximum information per unit of cost.

We will choose the values of n_r such that the minimum mean squared error for fixed cost function.

Suppose C_r and $C_{(y)}$ is the per unit cost for $\sum k_j$ auxiliary variables X_j and the study variable Y respectively. Suppose C_h is the overhead cost and C_t is the total cost, then a cost function may be defined as,

$$C_t = C_h + \sum_{r=0}^{m-1} C_r n_r + C_{(y)} n_m \tag{4.1}$$

The optimum choices of n_r (r = 1,2,...,m) for which mean sum of squares of errors of T_2 and T_3 are minimum for the cost function are given by

$$n_r^0 = \frac{1}{g\sqrt{C_r}} \left(C_t - C_h\right) \sqrt{\left(\rho^2_{y.12...k_m} - \rho^2_{y.12...k_r}\right)}$$

$$\text{and } n_m^0 = \frac{1}{g\sqrt{C_m}} \left(C_t - C_h\right) \sqrt{\left(1 - \rho^2_{y.12...k_m}\right)} \tag{4.2}$$

The minimum mean square error is

$$M_0 = \frac{n_0 \sigma_y^2}{n_0 - 1} \sum_{r=1}^{m} \left(\frac{g^2}{C_t - C_h} - \frac{1 - \rho^2_{y.12,...,k_1+k_2+....+k_r}}{n_0} \right) \tag{4.3}$$

where $g = \sum_{r=0}^{m-1} \sqrt{\left(\rho^2_{y.12,...,k_m} - \rho^2_{y.12,...,k_r}\right)} C_r + \sqrt{\left(1 - \rho^2_{y.12,...,k_m}\right)} C_m$

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